SOLUTIONS TO SELECTED PROBLEMS from THE PHYSICS OF RADIOLOGY

SOLUTIONS TO SELECTED PROBLEMS

from

The Physics of Radiology, Fourth Edition

By

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PREFACE

Many of our readers have told us that the numerous examples that form part of *The Physics of Radiology* are very helpful, but that there should be more of them. Although there are one hundred and twenty-eight worked out examples in the book, we still tend to agree with these comments.

It is a well established fact that one does not really understand physics until one can use it to solve problems. We have always encouraged teachers using our book to assign problems, and to make the solutions to these part of classroom discussions. This volume is intended to enhance that process. We have selected approximately half of the problems found at the end of each chapter and have given what we consider to be reasonable solutions to them. Our solutions include, where appropriate, discussion of assumptions that may have to be made, and where the relevant formulae and data are to be found. We have tried to explain the reasoning that we have used in arriving at our solutions, and in many cases, we have added "Comments" that are intended to show what we think are the important aspects of the problem.

When physical constants and other data are needed, we expect students to use, where possible, data from *The Physics of Radiology.* These will be found in the Appendix, or in tables or diagrams included in the chapters. We are well aware that many of the values so found are no longer the latest. Nevertheless, for the sake of consistency, we recommend that this be done. When values are required for quantities that are not listed in *The Physics of Radiology,* we recommend the use of any of the various editions of *The Handbook of Chemistry and Physics* (published by the CRC Press Inc., West Palm Beach, Florida USA).

We very frequently refer to equations, figures, tables, examples, and pages from *The Physics of Radiology.* For brevity, we do this without mentioning the text. When the reader encounters such otherwise unspecified references, it should be assumed that *The Physics of Radiology* is meant.

We have selected only about half of the problems. Numerous friends have pointed out, that if we provided answers to all of them, it would not be reasonable for teachers to assign problems from our book, and expect the students to work them out themselves. We make no claim as to the absolute correctness of our solutions, there is always room for improvement. We hope readers who disagree with them, or who feel they can be improved, will write to us. We also hope that all of our readers will benefit from our solutions and our discussions.

Finally, we would like to acknowledge help we have received in many forms from many people. Our own former students, with whom we have had many interesting discussions about the problems or the ideas behind them, are far too numerous to mention. So too are the many readers who have written to us pointing out errors or omissions in our text. They have all been a help to us and we are grateful to them. We also thank the many people who have written to us, informing us of errors that we have previously made in giving answers to some of the problems. We have taken these comments into account in arriving at our "solutions." On many occasions we have agreed with them, sometimes we have not.

We would like to mention a few people who have been of special help. We have had numerous conversations and consultations with Jake Van Dyk, Alan Rawlinson, and Phil Leung, all of the Ontario Cancer Institute, and they have been of special help to us. We thank Ian Cunningham of the Victoria Hospital and the J.P. Robarts Research Institute in London Ontario for providing answers and discussions for many of the problems in Chapter 16. Paul Johns, of Carleton University in Ottawa, has taken special care to provide improved or alternate solutions to problems in several of the chapters. Irv. Podgorsak and Monty Cohen of Montreal gave us more ideas then either of them have realized. May Day, of the Ontario Cancer Institute, typed much of the early drafts for us, and Chris Newcomb helped us with the conversion of this material to our own word processor. Of course, we also make special mention of our wives Sybil and Sheila who helped us tangibly with their support and patience when we were busy with yet another book.

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SOLUTIONS TO SELECTED PROBLEMS from THE PHYSICS OF RADIOLOGY

INTRODUCTION

In this book, solutions are given to about half of the problems found at the end of chapters 1 through 16 in The Physics of Radiology. These solutions include, where appropriate, discussion of assumptions that may be required, and where the relevant formulae and data are to be found. An attempt has been made to explain the reasoning used, and in many cases "Comments" have been added that are intended to indicate the relevence of the problem, and in some instances, connections with other problems or other topics in physics. Where appropriate, we have also added further discussion of aspects of the basic physics used in arriving at our solution.

The page numbers, where the problems are to be found in The Physics of Radiology, are given at the start of each chapter in this book. The statement of the problem, as it is in The Physics of Radiology, is given within a shaded box.

Referral is frequently made to equations, figures, tables, examples and pages from the Physics of Radiology. For brevity, this is done without mentioning the text. When the reader encounters such otherwise unspecified references, it should be assumed that the Physics of Radiology is meant.

When physical constants and other data are required, it is recommended that use be made of data from The Physics of Radiology where this is possible. These data are found in the Appendix, or in tables or diagrams included in the chapters. For values for quantities not found in the Physics of Radiology, the use of any of the various editions of the Handbook of Chemistry and Physics (published by the CRC Press Inc., West Palm Beach, Florida USA) is recommended.

Chapter 1

BASIC CONCEPTS

Selected Problems; pp. 34-36

1. A car accelerates at a rate of 5.0 km per hour per second. Express this acceleration in m s^{-2}.

We want the distance, which is given in km, to be expressed in meters, i.e. 1 km = 1,000 m: We also want the time expressed in seconds. 1 h = 3,600 s. There are two simple ways of handling this. We may simply replace km by 1,000 m and h by 3,600 s, which directly gives the result

$$a = 5.0\ \frac{km}{h\ s} = 5.0 \frac{(1000\ m)}{(3600\ s)s} = 1.39\ m\ s^{-2}$$

Another procedure is to note that, since 1 km = 1,000 m, we may divide by 1 km and multiply by 1,000 m without changing the meaning of the equation. Similarly with 1 h = 3,600 s. This gives

$$a = 5.0\ \frac{km}{h\ s} = 5.0\ \frac{km}{h\ s}\ x \left(\frac{1000\ m}{1\ km}\right) x \left(\frac{1\ h}{3600\ s}\right) = 1.39\ m\ s^{-2}$$

Comment: This problem is an exercise in manipulating units as well as numbers. Each quantity, such as 5.0 km/h s, is a combination of a number and one or more units. The units, like the numbers, obey the rules of algebra and can be multiplied and divided, and hence cancelled. Multiplication, combined with division by an equal value does not alter a quantity. 1,000 m is equal to 1 km, hence multiplication by 1,000 m/1 km changes only the units. Similarly for 1 hr/3,600 s. Our second method appears somewhat more cumbersome than does the first, but some physicists believe that it is less likely to introduce confusion when the combinations of the units are more complicated than they are in the above example.

2. A current of 1.0 μA flows into a condenser of capacity of 100 nF for 5.0 s. Calculate the potential difference between the plates of the condenser.

As current flows into the condenser, the voltage across it increases proportionately. The relation between voltage and charge is

$$V = Q/C \qquad \text{(Eq. 1-11)}^*$$

[1]where V is the voltage, Q is the charge on the condenser and C is the capacity of the condenser. The capacity is 100 nF (= 100 x 10^{-9} F) and the charge accumulates at a steady rate

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* Equation references are to the Physics of Radiology, 4th Edition

of 1.0 μA for 5.0 s, then

$$Q = V \times C = (1.0\ \mu A) \times (5.0\ s) = 5.0\ \mu A\ s = 5.0 \times 10^{-6}\ A\ s = 5.0 \times 10^{-6}\ C$$

and therefore the voltage, or potential difference, between the plates of the condenser, after 5 seconds is

$$V = \frac{Q}{C} = \frac{5.0 \times 10^{-6}\ C}{100 \times 10^{-9}\ F} = 50\ volt$$

3. A patient is given an x ray exposure of 200 R. Calculate this exposure in C kg^{-1}.

By the definition of the roentgen (see Page 10);

$$1\ R = 2.58 \times 10^{-4}\ C\ kg^{-1}$$

Therefore, the exposure is

$$200\ R = 200\ R \times \frac{2.58 \times 10^{-4}\ C\ kg^{-1}}{1\ R} = 516 \times 10^{-4}\ \frac{C}{kg}$$

$$= 0.0516\ C/kg$$

Comment: This problem is intended to reinforce the original meaning of the quantity exposure. Exposure was originally expressed in terms of the amount of electric charge released in a small volume of air. The modern meaning of exposure focuses more on the energy transferred from the radiation to the air. In fact, the ICRU recommends that exposure as a quantity should cease to be used. It is to be replaced by "air kerma", which gives directly the energy transferred from the radiation to the air (see Section 7.02).

7. Lithium in nature consists of two isotopes of atomic masses 6.01513 and 7.01601, with percentage abundances of 7.42 and 92.58%. Find the atomic weight. Check your answer with Table 1-3.

Atomic mass of $^{6}_{3}Li$ = 6.01513, % = 7.42

Atomic mass of $^{7}_{3}Li$ = 7.01601, % = 92.58

Atomic weight = (0.0742 x 6.01513) + (0.9258 x 7.01601) = 6.94174

For comparison with Table 1-3, this should be rounded off to 4 significant figures, which would be 6.942. Table 1-3 lists 6.941.

8. In the complete decay of 1000 atoms of $^{12}_{5}B$, how many beta particles will be released and how much energy as gamma rays? (See Figure 1-3).

The modes of decay of $^{12}_{5}B$ are given in Figure 1-3. It is shown that a beta particle is emitted with no gamma ray for 98% of the transitions. In the other 2%, when a beta particle is emitted it is followed by a gamma ray of energy 4.4 MeV. Thus, for the decay of 1000 atoms, 1000 beta particles will be emitted but only 20 gamma rays. The energy released $E_{h\nu}$, as gamma radiation would thus be

$$E_{h\nu} = 20 \ x \ 4.4 \ MeV = 88 \ MeV$$

11. A radio station transmits at a frequency of 900 kHz. Find the wavelength of the radiation. Find the energy in electron volts of 1 quantum of such radiation.

The wave length λ, can be calculated using Equation 1-14;

$$\nu \lambda = c \qquad \text{(Eq. 1-14)}$$

where

c	= the speed of light	= 2.998 x 10^8 m/s		(Table A-1)
ν	= the frequency	= 900 x 10^3 s^{-1}	= 900 kHz	

therefore

$$\lambda = \frac{c}{\nu} = \frac{2.998 \ x \ 10^8 \ m \ s^{-1}}{900 \ x \ 10^3 \ s^{-1}} = 333 \ m$$

The energy of a quantum of this radiation would be

$$E = h \ \nu \qquad \text{(Eq. 1-15)}$$

where h is Planck's constant (= 6.63 x 10^{-34} J s, from Table A-1), then

$$E = (6.63 \ x \ 10^{-34} \ J \ s) \ x \ (900 \ x \ 10^3 \ s^{-1}) = 5.97 \ x \ 10^{-28} \ J$$

Since 1 eV = 1.602 x 10^{-19} J (Table A-1), the energy in electron volts is

$$E = 5.97 \ x \ 10^{-28} \ J \ x \ \frac{1 \ eV}{1.602 \ x \ 10^{-19} \ J} = 3.73 \ x \ 10^{-9} \ eV$$

Comment: This is an exceedingly small amount of energy. Normally, for radiation of such long wavelengths, the quantum nature of the radiation is not useful in describing the way the radiation interacts with matter. The wavelength tells us, for example, how long an antenna should be to transmit or receive such waves. For maximum effect, the antenna should be about one quarter of the wavelength or 333/4 m ≈ 80 m. This frequency is in the AM radio band and transmitter antennae are of approximately this size.

12. Electromagnetic waves of length 10 cm are used in radar installations. Calculate the frequency of such radiation and the energy of 1 quantum of such radiation.

The relation between frequency and wavelength is given by Equation 1-14;

$$\nu\lambda = c \qquad \text{(Eq. 1-14)}$$

The wavelength λ, is 10 cm. Therefore the frequency ν, is

$$\nu = \frac{c}{\lambda} = \frac{2.998 \ x\ 10^{8}\ m\ s^{-1}}{0.1\ m} = 2.998\ x\ 10^{9}\ s^{-1} \approx 3.0\ GHz$$

The energy of one quantum of this radiation is given by Equation 1-15 (see also Problem 11), so

$$E = h\nu = (6.63\ x\ 10^{-34}\ J\ s)\ x\ (3.0\ x\ 10^{9}\ s^{-1}) = 19.89\ x\ 10^{-25}\ J$$

or

$$E = 19.89\ x\ 10^{-25}\ J\ x\ \frac{1\ eV}{1.602\ x\ 10^{-19}\ J} = 12.4\ x\ 10^{-6}\ eV$$

Comment: The energy of a quantum of this radiation is still very small, and is very much less than is required to tear electrons away from atoms or molecules. This energy is typically of the order of the order of one to a few hundredths of one electron volt. In fact it is even much less than the kinetic energy possesed by a molecule of a gas at room temperature. This kinetic energy can easily be calculated from the equation

$$K.E._T = \frac{2}{3} kT$$

where k is Boltzmann's constant and T is the absolute temperature. This equation is found in most basic physics texts, a good example of which is 'Physics' by Halliday and Resnick, John Wiley and Sons, Inc. k is 1.38 x 10^{-23} J/K and we may take room temperature to be 22°C, T = 273 + 22 = 295 K. Thus the kinetic energy is

$$K.E._T = \frac{2}{3}\ x\ 1.38\ x\ 10^{-23}\ \frac{J}{K}\ x\ 295\ K = 271\ x\ 10^{-23}\ J$$

$$= 271\ x\ 10^{-23}\ J\ x\ \frac{1\ eV}{1.602\ x\ 10^{-19}\ J} \approx 17\ x\ 10^{-3}\ eV$$

which is almost 1,000 times the quantum energy. For this reason such radiation is called "non-ionizing radiation". It is one of the reasons why it would be quite surprising if radio waves are found to produce harmful effects in tissues. Medical linear accelerators use waves of approximately this length to accelerate electrons in their acceleration tubes.

16. Determine the minimum wavelength of the radiation emitted from a 35 MeV linac.

A 35 MeV linac would produce radiation with a minimum wavelength λ, that can be calculated using Equation 1-16.

$$E = \frac{1240\ keV\ pm}{\lambda} \qquad \text{(Eq. 1-16)}$$

Therefore

$$\lambda = \frac{1240\ keV\ pm}{E} = \frac{1.240\ MeV\ pm}{35.\ MeV} = 0.0354\ pm = 35.4\ x\ 10^{-15}\ m$$

Comment: This wavelength is very much less than the size of any molecule or atom. Matter "looks" like empty space to this radiation and in general such radiation will pass through it readily.

17. A radio transmitter operates at 10 kW at 300 kHz. Find the number of quanta emitted per second.

The energy of one quantum of such radiation is obtained by using Equation 1-15

$$E = h\nu \qquad \text{(Eq. 1-15)}$$

From Appendix A-1, Planck's constant, h = 6.63 x 10^{-34} J s, therefore

$$E = (6.63\ x\ 10^{-34}\ J\ s)\ x\ (300\ x\ 10^{3}\ s^{-1}) = 199\ x\ 10^{-30}\ J$$

The power is 10 kW = 10 x 10^3 W = 10^4 J s^{-1}, and the number N, of quanta emitted per second is

$$N = \frac{10^4\ J\ s^{-1}}{199\ x\ 10^{-30}\ J} = 50.3\ x\ 10^{30}\ s^{-1}$$

Comment: The calculation we have made here would describe the **activity** of the transmitter as a source of radiation. The units of this result are s^{-1}. Although these are the same as those used for the frequency of the radiation, the meaning is quite different. Frequency is a periodic phenomenon and is a property of the radiation itself, while the emission of quanta is not periodic and is a property of the emitter and not of the radiation. This is the reason that different **special units** have been assigned to them. The special unit for frequency is the hertz and for activity it is the becquerel.

19. An antiproton is a proton with a negative charge. What energy would be emitted if a proton annihilated an antiproton?

The mass of a proton is given in Problem 9 as 1.007276 atomic mass units (amu). The mass of an antiproton has the same value.

In Example 1-7, the energy equivalent to 1 amu is shown to be 931.6 MeV.

The energy released by the annihilation of a proton and an antiproton is thus,

$$E = 2\ x\ (1.007276\ amu)\ x \left(\frac{931.6\ MeV}{1\ amu}\right) = 1{,}876\ MeV = 1.876\ GeV$$

21. Show that when $v << c$, Equation 1-21 reduces to K.E. = $m_o v^2/2$.

Using equation 1-21 we can calculate the relativistic kinetic energy of a mass m_o

$$K.E. = mc^2 - m_o c^2 \qquad \text{(Eq. 1-21)}$$

If the velocity is v, the mass can be calculated using Equation 1-20

$$m = \frac{m_o}{\sqrt{1 - v^2/c^2}} \qquad \text{(Eq. 1-20)}$$

When this is substituted into Equation 1-21, we have for the kinetic energy

$$K.E. = mc^2 - m_o c^2 = \frac{m_o c^2}{\sqrt{1 - v^2/c^2}} - m_o c^2 = m_o c^2 \left[(1 - v^2/c^2)^{-1/2} - 1 \right]$$

The quantity $(1 - v^2/c^2)^{-1/2}$ can be expressed as a series of terms of increasing powers of v^2/c^2 by using the binomial expansion. The first three terms of the binomial expansion of $(1 - x)^n$ are

$$(1 - x)^n = 1 - nx + \frac{n(n-1)}{2!} x^2 - .....$$

(For an explanation of this equation, see almost any text on elementary calculus)

Substituting $x = v^2/c^2$, and $n = -1/2$, we have for these three terms

$$\left(1 - \frac{v^2}{c^2}\right)^{-1/2} = 1 - \left(-\frac{1}{2}\right)\frac{v^2}{c^2} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(\frac{v^2}{c^2}\right)^2 - ....$$

Because v is much smaller than c, each term is much smaller than the previous one, and we may neglect all but the first two, which gives for the kinetic energy

$$K.E. \approx m_o c^2 \left(1 + \frac{v^2}{2c^2} - 1\right) = \frac{1}{2} m_o v^2$$

Comment: Equations 1-20 and 1-21 come directly from the Special Theory of Relativity, and the fact that the above result reduces to the familiar expression for the kinetic energy when $v << c$, illustrates that Relativity did not overthrow Newton's laws but rather extended them.

24. The gamma rays from ^{60}Co have a half-value layer in lead of 1.1 cm. Estimate the thickness of lead required to attenuate such a beam by a factor of 10^6. What is the mean range of these photons in lead?

The term "half-value layer" has been used in phrasing the question, and this implies that we should assume exponential attenuation.

From Equation 1-37

$$N = N_0 e^{-\mu x} \qquad where \quad \mu = 0.693/x_h$$