TRAFFIC ACCIDENT INVESTIGATORS' AND RECONSTRUCTIONISTS' BOOK OF FORMULAE AND TABLES

.

SECOND EDITION

TRAFFIC ACCIDENT INVESTIGATORS' AND RECONSTRUCTIONISTS' BOOK OF FORMULAE AND TABLES

By

R. W. RIVERS

Inspector Traffic Branch Royal Canadian Mounted Police (Retired)



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INTRODUCTION

This book contains a summary of formulae and tables commonly required and used in traffic and investigation and reconstruction. In many cases, alternative problem-solving formulae to accommodate various known data are presented. These formulae also provide a means of checking the accuracy of one answer against others using the alternative data and alternate approaches available in problem-solving.

To use this manual in problem-solving, the investigator should decide upon what is the problem or what is to be determined, e.g., a speed determination (S), a time factor (t), and so on. To arrive at or to calculate the answer, the investigator should then determine what information is available to assist in finding the answer. Once this information is decided upon, the investigator should then go to the specific chapter covering the problem to be answered, such as the Slide-to-a-Stop Speed chapter. The symbol relating to what must be solved will always be on the left side of the formulae, generally to the left side of the equal (=) sign. An example is the speed from skid marks formula: $S = \sqrt{30} df$. Here the problem to be solved is the speed of an accident vehicle based on skid marks. The symbols to the right (very often inside a radical) indicate the information that must be known in order to calculate the answer. In this example, the distance or length of skid (d) and the drag factor (f) must be known in order to calculate the speed (S).

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THE METRIC (S.I.) SYSTEM

The metric system, called SI, is from the French name Le Système International d'Unites. Most countries outside the United States use the metric system (S.I.), but practitioners in these countries very often use reference materials published in the United States. Continuing with the precedent established by the first edition, this work is prepared for use on an international basis. That is to say, all mathematical formulae and problem-solving examples are shown in both the United States/Imperial or English and Metric (S.I.) systems.

In North America, a decimal fraction is generally indicated by means of a (decimal) point on the line (not a dot in the raised or centered position). In this regard, caution is important for North Americans and many others in reading the various literature that in some countries it is the dot in the raised position that is used, and in some countries it is the comma. The North American practice of using the dot as a decimal point situated on the line is followed in this manual.

To assist the reader and to avoid confusion, precise conversions from one system to the other, conversion factors and constants will be found throughout the book, particularly in the appendices where specific conversion tables will be found.

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DISCLAIMER

Many published books and technical papers have been stud-ied and participation in many field tests made in the preparation of this book of formulae and tables. The information and practices set out herein are, to best of the author's knowledge, experience, and belief, the most current and accurate in the traffic accident investigation and reconstruction profession. However, the author, publisher, and editors expressly disclaim all and any liability to any person, whether a purchaser of this publication or not, as a consequence of anything stated, done or omitted to be done, whether in whole or in part by such person in reliance upon any part of the contents of this publication. Every acceptable procedure may not be presented herein, and some of the circumstances of a given case may require additional or substitute procedures. Also, since statutes, ordinances, and organizational policies and procedures differ widely in various jurisdictions, those of the particular jurisdiction should govern when there is any conflict between them and the contents of this book.

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TRAFFIC ACCIDENT INVESTIGATORS' AND RECONSTRUCTIONISTS' BOOK OF FORMULAE AND TABLES

Chapter 1

SYMBOLS

T he following table provides a ready reference of symbols, with descriptions, commonly used in traffic accident investigation and reconstruction. These symbols and definitions will assist the reader in gaining a better understanding of the more complex formulae found in the text, as well as those used in other traffic accident investigation and reconstruction textbooks.

TABLE 1-1

GENERAL SYMBOLS, DEFINITIONS AND MEASUREMENT UNITS

Symbols and Descriptions	Measurement Units	
	<i>U.S</i> .	<i>S.I.</i>
General		
$a = acceleration (\pm)$	ft/s²	m/s²
C = chord	ft, in	m, cm
CG = center of gravity	ft, in	m, cm
CM = center of mass	ft, in	m, cm
d = distance, displacement	ft, in	m, cm
e = superelevation	percent	percent
(See also lower case m)	-	-
f = drag factor	decimal fraction	decimal fraction
$\mu = \text{coefficient of friction}$	decimal fraction	decimal fraction
$f = acceleration factor (\pm)$	Usually expressed	Usually expressed
	as a decimal	as a decimal
	fraction	fraction

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$f_a = acceleration factor$	Usually expressed as a decimal	Usually expressed as a decimal	
f_d = deceleration factor	Usually expressed as a decimal	Usually expressed as a decimal fraction	
$f_L = lateral acceleration$	Usually expressed as a decimal fraction	Usually expressed as a decimal fraction	
$\mathbf{F} = \mathbf{force}$ in			
pounds (newtons)	lbf	N kef	
g = acceleration	32.2 ft/s^2	9.81 m/s^2	
due to gravity			
h = height	ft, in	m, cm	
I = inertia	slugs	kg, grams	
kg = kilograms	Ŭ	0, 0	
Ke = kinetic energy			
L = length	ft, in	m, cm	
l = length	ft, in	m, cm	
M = mass	lb	kg	
M = middle ordinate	ft, in	m, cm	
Mom = momentum	lb.ft/s	kg.m/s	
p = momentum	lb.ft/s	kg.m/s	
m = grade, slope (See also lower case e)	percent	percent	
n = percentage factor	E.g., percent	E.g., percent	
1 0	braking	braking	
	efficiency	efficiency	
R = radius	ft, in	m, cm	
r = run, rise or fall	ft, in	m, cm	
(Used in measuring grade, slope or superelevation)			
S = speed	mph	km/h	
t = time	hr, min, sec	hr, min, sec	
$t_w = track width$	Usually expressed	Usually express	
	in inches	in cm	
Mu = Coefficient of	Usually expressed	Usually expressed	
friction	as a decimal	as a decimal	
(See also <i>f</i> , drag factor)	fraction	fraction	
V = velocity	ft/s	m/s	
W = weight	lb	kg	

Symbols

Symbols to Denote Specific Angles

$\alpha = alpha$	Angle in degrees
$\theta = theta$	Angle in degrees
$\phi = phi$	Angle in degrees
$\mathbf{Y} = \mathbf{psi}$	Angle in degrees
$\Omega = omega$	Any other specific angle

Common Symbols in Solving for Momentum Speed

$\alpha = alpha$	Vehicle 1's approach angle
$\theta = theta$	Vehicle 1's departure angle
$\phi = phi$	Vehicle 2's departure angle
$\mathbf{Y} = \mathbf{psi}$	Vehicle 2's approach angle
$\Omega = omega$	Any other specific angle

General Symbols in Traffic Accident Reconstruction

tan = tangent $\tan \Theta = \text{tangent of angle } \Theta$ \in = coefficient of restitution Δ = delta (Used to indicate change, e.g., Δ V = change in velocity) ∞ =directly proportional to \cong = approximately equal; congruent $\sim =$ similar to; equivalent $\infty = infinity$ $\pi = pi$ (3.14159) The ratio of the circumference and a diameter of the same circle. $\sqrt{1}$ = radical sign $\Sigma =$ sigma; summation of $\pm =$ plus or minus $\cos = \cos ine$ $\sin = \sin e$ tan = tangentn = ad infinitum \therefore = therefore W = workft-lb N-m or j Y = Y axis; vertical axis

X = X axis; horizontal axis

Subscripts

- X_0 = subscript o denoting the original or initial factor for the component to which it is attached.
- X_7 = subscript 1 denoting an initial factor, such as speed or velocity, or number 1 for the component to which it is attached when two or more factors are involved. Additional factors may be denoted by subscripts 2 and 3 and so on.
- X_{f} = subscript f denoting the final factor for the component to which it is attached.

Overline

 \overline{S} = average speed. The small bar (overline) denotes average for the component to which it is attached.

Chapter 2

ACCELERATION

DEFINITION

Acceleration and deceleration are the rates of speed or velocity change per unit of time, usually measured in feet per second per second $(ft/s/s \text{ or } ft/s^2)$ or meters per second per second $(m/s/s \text{ or } m/s^2)$. Acceleration is an increase (+) and deceleration is a decrease (-) in speed or velocity.

ACCELERATION DUE TO GRAVITY

Formula 2-01

А.	$g = \frac{GM}{d^2}$	
В.	U.S. g = 32.2 ft/s ²	<i>S.I.</i> 9.81 m/s ²
where $g = acceleration$ due to gravity G = gravitational constant		

M = mass of earth

d = distance from center of earth

ACCELERATION FACTOR

An *acceleration factor* is a variable or unit describing the time rate of change of velocity. The factor can be used in either a pos-

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itive (+) or a negative (-) sense, depending upon the circumstances in which it is used. In a positive sense, the result is known as acceleration; in a negative sense as negative acceleration, appropriately referred to as being and understood to be *deceleration*.

Acceleration Factor for a Vehicle that Accelerates from a Lower Speed or Velocity to a Higher Speed or Velocity During a Known Time

Formula 2-02

$$U.S. S.I. S.I.$$

V_f = final velocity t = time $S_o = initial speed$ $\hat{S_f} = \text{final speed}$ $V_o = \text{initial velocity}$

Deceleration Factor for Vehicle that Slows from a Higher Speed to a Lower Speed Over Known Distance

Formula 2-03

$$\begin{aligned} U.S. & S.I. \\ f_{d} &= \frac{S_{o}^{2} - S_{f}^{2}}{30 \text{ d}} & f_{d} &= \frac{S_{o}^{2} - S_{f}^{2}}{254 \text{ d}} \end{aligned}$$

where f_d = deceleration factor $S_o = initial speed$ $S_f = final speed$

$$d = distance$$