

THE FUNDAMENTALS OF IMAGING PHYSICS AND RADIOBIOLOGY



Copy of Roentgen's radiograph of his wife's "hand with rings," made soon after his discovery of x rays in November 1895 in Wurzburg, Germany. (See Glasser O. *Dr. W. C. Roentgen*, p. 39, 2nd ed., Springfield, Charles C Thomas, 1958.)

NINTH EDITION

**THE FUNDAMENTALS
OF
IMAGING PHYSICS
AND
RADIOBIOLOGY**

By

JOSEPH SELMAN, M.D., FACR, FACP

*Medical Advisor
Radiation Therapy Center
East Texas Medical Center Cancer Institute*



Charles C Thomas
PUBLISHER • LTD.
SPRINGFIELD • ILLINOIS • U.S.A.

Published and Distributed Throughout the World by

CHARLES C THOMAS • PUBLISHER

2600 South First Street
Springfield, Illinois 62794-9265

This book is protected by copyright. No part of
it may be reproduced in any manner without
written permission from the publisher.

© 2000, 1994, 1985, 1977, 1972, 1965, 1961, 1957, and 1954
by CHARLES C THOMAS • PUBLISHER

First Edition, 1954
Second Edition, 1957
Third Edition, 1961
Fourth Edition, 1965
Fifth Edition, 1972
Sixth Edition, 1977
Seventh Edition, 1985
Eighth Edition, 1994
Ninth Edition, 2000

ISBN 0-398-06987-5

Library of Congress Catalog Card Number: 99-035813

With THOMAS BOOKS careful attention is given to all details of manufacturing and design. It is the Publisher's desire to present books that are satisfactory as to their physical qualities and artistic possibilities and appropriate for their particular use. THOMAS BOOKS will be true to those laws of quality that assure a good name and good will.

Printed in the United States of America
SM-HP-3

Library of Congress Cataloging-in-Publication Data

Selman, Joseph.

The fundamentals of imaging physics and radiobiology / Joseph Selman --
9th ed.

p. cm.

Prev. ed.: The fundamentals of x-ray and radium physics.

Includes bibliographical references and index.

ISBN 0-398-06987-5

1. X-rays. 2. Radium. 3. Radiography. I. Selman, Joseph.
Fundamentals of x-ray and radium physics. II. Title.

QC481 .S456 2000

537.5'35--dc21

99-035813

RW

Dedicated to my wife

PREFACE TO THE NINTH EDITION

A number of important changes have been made in this edition, the first being the obvious new title reflecting the demise of radium in medicine and its replacement by artificial radioisotopes (radionuclides).

To improve readability, a double-column format has been adopted, serving to bring more illustrations closer to their explanation in the text. A helpful addition should be the outline of the contents of each chapter below its heading.

Greater attention is paid to high-frequency generation of x rays. Because this involves microprocessors (computers), which also find wide application in the department generally, I have added a chapter on basic computer science. A typical modern control panel for comparison with the primitive type indicates how much progress has occurred in x-ray production and control.

The rapid increase in the number of approved mammography centers demands a corresponding need for specialized technologists. I have therefore updated the requirements for mammographic equipment and included a section outlining quality assurance, especially as it applies to the responsibilities of the designated mammographer. A separate chapter is now devoted to mammography.

Digital imaging has been assigned a new chapter to include more information about subtraction angiography.

A new chapter takes up the basic science of radiobiology: the effects of radiation, especially of x rays, on cells, tissues, and organs. Since harmful effects are dose-

dependent, this fits well with the chapter on health physics, which has been updated.

More questions and problems, with sample solutions, have been added. Besides, the index continues to be user-friendly to facilitate the search for answers in the text.

Many thanks are due Joe Burrage (Diagnostic Imaging, Inc.) for help with modern x-ray generator circuitry in the form of block diagrams, as well as explanations of newer equipment and devices. I am also grateful to Ted Kosnik, Ph.D., for his answers about matters physics, and for plotting the attenuation curves of 100-kV x rays; and to my son Jay E. Selman, M.D., for his review of the chapter on computer science and his very helpful comments.

I am also indebted to the following individuals and companies, thanking them for providing brochures and technical information on equipment and supplies: Larry Spittler, RT (Diagnostic Imaging, Inc.); Nu-Tech (Tyler, Texas); and Paul Oster, RT, CNMT, Chief Technologist of the Nuclear Medicine Laboratory, East Texas Medical Center Hospital, Tyler, Texas.

New illustrations and modifications of previous ones have been meticulously executed by artist Gene Johnson (Tyler, Texas), and for this I am most grateful. At this point I wish to thank again the retired artist, Howard Marlin, for his excellent illustrations in the first eight editions of this book, dating back to 1954, and carried over to the ninth edition.

Finally, and by no means least, I would like to express my sincere gratitude to Michael Thomas, head of Charles C Thomas, Publisher, and his most competent

staff, and to Claire Slagle, the superb editor of my book for its Ninth Edition.
assigned to me, for their cooperation and
diligent attention to details in the publication

Joseph Selman, M.D.

INTRODUCTION

As a student entering an approved School of Radiologic Technology, you have probably heard about the hazards of overexposure to x rays, a special form of light. The key word here is “approved,” a designation assuring that the school has in place not only the required teaching program, but also all necessary protective measures to keep your occupational exposure to x rays and related radiation within acceptable limits. Necessary protective maintenance has been established by regulations (“regs”) mandated by the relevant State Agency and the U.S.A. Nuclear Regulatory Commission (NRC). I shall briefly summarize here the basic protective features that you will find in a modern radiology department.

The walls of the x-ray rooms have built-in lead or equivalent protection in accordance with State regulations, and have been tested by a certified Health Physicist or other qualified person. A number of protective features have been incorporated in the x-ray equipment as mandated by the Bureau of Radiological Health (BRH). You will note that the console, which controls x-ray tube operation by the radiographer, is located inside a control booth whose walls contain a prescribed thickness of lead (metal) for the radiographer’s protection; even the booth’s window consists of protective glass of proper

thickness.

In fluoroscopy you will see a lead-containing rubber or plastic curtain hanging from the viewing assembly to protect the fluoroscopist; leaded aprons and gloves are also available for your personal protection.

The radiographer must follow all rules governing radiation protection according to the three basic principles: distance, shielding, and time: radiation dose decreases rapidly with increasing distance from the x-ray source; shielding of the hands and body by lead or other protective material; and reducing the time spent in the area during exposure all contribute greatly to dose reduction. Finally, special badges must be worn by all personnel in the x-ray area to detect any breach in safe operating procedures.

In my experience over many years, monitored exposures of radiographers have, with rare exceptions, been well within prescribed limits. Moreover, the risk from occupational exposure to radiation in the radiology department is trivial compared to other occupational, recreational, and highway travel risks. The same applies to the risk in a modern radiology department.

The chapter on Health Physics, which you will study later, provides additional details on the subject of radiation protection of personnel and patients.

CONTENTS

	<i>Preface</i>	<i>Page</i> vii
	<i>Introduction</i>	ix
	<i>Chapter</i>	
1.	SIMPLIFIED MATHEMATICS	3
	Arithmetic	3
	Fractions	3
	Percent	5
	Decimal Fractions	5
	Significant Figures	5
	Algebra	6
	Ratio and Proportion	9
	Plane Geometry	10
	Similar Triangles	11
	Graphs and Charts	12
	Large and Small Numbers	13
	Logarithms	14
	Questions and Problems	15
2.	PHYSICS AND THE UNITS OF MEASUREMENT	16
	Standard Units	17
	Fundamental Units	17
	Derived Units	18
	Manipulation of Units	19
	Prefixes Applied to SI Units	21
	Questions and Problems	21
3.	THE PHYSICAL CONCEPT OF ENERGY	22
	Force	22
	Work and Energy	22
	Law of Conservation of Energy	23
	Questions and Problems	25
4.	THE STRUCTURE OF MATTER	26

Subdivisions of Matter	26
Atomic Structure—the Electrical Nature of Matter	28
Atomic Number	30
Mass Number	30
Isotopes and Nuclides	30
The Periodic Table	31
Chemical Behavior	34
Ionization	36
Questions and Problems	38
5. ELECTROSTATICS	39
Definition	39
Electrification	39
Methods of Electrification	39
Laws of Electrostatics	41
Electroscope	43
Static Discharge	44
Questions and Problems	45
6. ELECTRODYNAMICS—ELECTRIC CURRENT	46
Definition	46
The Nature of an Electric Current	46
Sources of Electric Current	47
Factors in an Electric Circuit	47
Ohm's Law	50
Cells and Batteries	50
Components of Elementary Electric Circuits	51
Series and Parallel Circuits	52
Electric Capacitor (Condenser)	56
The Work and Power of a Direct Current	57
Questions and Problems	58
7. MAGNETISM	59
Definition	59
Classification of Magnets	59
Laws of Magnetism	60
Nature of Magnetism	60
Magnetic Fields	61
Characteristics of Lines of Force	62
Magnetic Induction (Magnetization)	63

	Magnetic Permeability and Retentivity	64
	Magnetic Classification of Matter	64
	Earth's Magnetism	64
	Questions and Problems	65
8.	ELECTROMAGNETISM	66
	Definition	66
	Electromagnetic Phenomena	66
	The Electromagnet	67
	Electromagnetic Induction	68
	Direction of Induced Electron Current	69
	Self-induction	69
	Mutual Induction	71
	Questions and Problems	71
9.	ELECTRIC GENERATORS AND MOTORS	72
	Electric Generator	72
	Definition	72
	Essential Features	72
	Simple Electric Generator	73
	Properties of Alternating Currents	75
	Direct Current Generator	76
	Advantages of Alternating Current	77
	Electric Motor	78
	Definition	78
	Principle	78
	Simple Electric Motor	79
	Types of Electric Motors	79
	Current-measuring Devices	81
	Questions and Problems	82
10.	PRODUCTION AND CONTROL OF HIGH VOLTAGE– REGULATION OF CURRENT IN THE X-RAY TUBE	83
	Transformer	83
	Principle	83
	Construction of Transformers	85
	Efficiency and Power Losses	86
	Control of High Voltage	88
	Autotransformer	88
	Control of Filament Current and Tube Current	89

	Choke Coil	89
	Rheostat	90
	High-Frequency Control of Current and Voltage	91
	Questions and Problems	92
11.	RECTIFICATION AND RECTIFIERS	94
	Definition	94
	Methods of Rectifying an Alternating Current	95
	Rectifiers	100
	Rectifier Failure	105
	Questions and Problems	106
12.	X RAYS: PRODUCTION AND PROPERTIES	108
	How X Rays Were Discovered	108
	Nature of X Rays	109
	Source of X Rays in Radiology	111
	The X-Ray Tube	111
	Details of X-Ray Production	112
	Electron Interactions with Target Atoms—X-ray Production	113
	Target Material	115
	Efficiency of X-ray Production	115
	Properties of X Rays	115
	Specifications of the Physical Characteristics of an X-ray Beam	116
	X-ray Exposure (Quantity)	116
	X-ray Quality	117
	“Hard” and “Soft” X Rays	123
	The Interactions of Ionizing Radiation and Matter	124
	Relative Importance of Various Types of Interaction	129
	Detection of Ionizing Radiation	130
	Summary of Radiation Units	131
	Exposure—the Roentgen (R)	131
	Absorbed Dose—the Gray (Gy)	131
	Modification of Kilovoltage X-ray Beams by Filters	132
	Questions and Problems	133
13.	X-RAY TUBES	135
	Thermionic Diode Tubes	135
	Radiographic Tubes	136
	Glass Envelope	136
	Cathode	136

	Anode	138
	Space Charge Compensation	141
	Factors Governing Tube Life	143
	Questions and Problems	148
14.	X-RAY CIRCUITS	149
	Source of Electric Power	149
	Main Single-Phase X-Ray Circuits	150
	Completed Wiring Diagram	158
	Basic X-Ray Control Panel or Console	158
	Three-Phase Generation of X-Rays	162
	High-Frequency Generation of X Rays	165
	Power Rating of X-Ray Generators and Circuits	166
	Falling-Load Generator	167
	Special Mobile X-Ray Equipment	168
	Battery-Powered Mobile X-Ray Units	168
	Capacitor (Condenser)-Discharge Mobile X-Ray Units	168
	Questions and Problems	171
15.	X-RAY FILM, FILM HOLDERS, AND INTENSIFYING SCREENS	173
	Composition of X-Ray Film	173
	Types of Films	175
	Practical Suggestions in Handling Unexposed Film	176
	Film Exposure Holders	176
	Intensifying Screens	177
	Questions And Problems	186
16.	THE DARKROOM	188
	Introduction	188
	Location of the Darkroom	188
	Building Essentials	189
	Entrance	189
	Size	190
	Ventilation	190
	Lighting	190
	Film Storage Bin	191
	Questions And Problems	191
17.	CHEMISTRY OF RADIOGRAPHY AND FILM PROCESSING	192

	Introduction	192
	Radiographic Photography	192
	Radiographic Chemistry	193
	Manual Processing	195
	Film Fog, Stains, and Artifacts	196
	Automatic Processing	197
	Summary of Processor Care	203
	Silver Recovery from Fixing Solutions	203
	Questions And Problems	203
18.	RADIOGRAPHIC QUALITY	205
	Blur	206
	Geometric or Focal Spot Blur	206
	Focal Spot Evaluation	208
	Motion Blur	210
	Screen Blur	210
	Object Blur	212
	Radiographic Density	213
	Contrast	217
	Radiographic Contrast	217
	Subject Contrast	218
	Film Contrast	220
	Distortion	222
	Direct Magnification or Enlargement Radiography (Macroradiography)	226
	Modulation Transfer Function	228
	Questions and Problems	230
19.	DEVICES FOR IMPROVING RADIOGRAPHIC QUALITY	232
	Scattered Radiation	232
	Removal of Scattered Radiation By a Grid	234
	Principle of the Radiographic Grid	234
	Efficiency of Grids	235
	Types of Grids	238
	Precautions in the Use of Focused Grids	240
	Practical Application of Grids	242
	Removal of Scattered Radiation by an Air Gap	246
	Reduction of Scattered Radiation by Limitation of the Primary Beam	247

Other Methods of Enhancing Radiographic Quality	252
Moving Slit Radiography	252
The Anode Heel Effect—Anode Cutoff	254
Compensating Filters	254
Summary of Radiographic Exposure	255
Questions and Problems	255
20. FLUOROSCOPY	258
The Human Eye	258
Fluoroscopic Image Intensification	259
Magnification in Image Intensifier	263
Multiple-Field Intensifiers	263
Viewing the Fluoroscopic Image	264
Optical Lens System	264
Television Viewing System	264
Video Cameras	264
Television Monitor	266
Charge-Coupled Device (CCD) TV Camera	267
Quality of the TV Image	267
Recording the Fluoroscopic Image	268
Recording the Video Image	269
Laser Discs	270
Questions and Problems	271
21. VANISHING EQUIPMENT	273
Stereoscopic Radiography	273
Tomography	276
Xeroradiography (Xerography)	282
Questions and Problems	284
22. MAMMOGRAPHY	285
Quality Standards in Mammography	289
Qualifications of Radiologic Technologist (Mammographer)	291
Equipment	291
Quality Assurance	291
Quality Assurance Records	291
Radiologic Technologist Responsibilities	291
Quality Control Tests	292
Daily	292
Weekly	292

	Monthly	292
	Quarterly	293
	Semiannually	293
	Mobile Mammography Units	294
	Questions and Problems	294
23.	BASIC COMPUTER SCIENCE	295
	Introduction	295
	History	295
	Data <i>vs</i> Information	297
	Computer Operations	297
	Computer Components	298
	Computer Language	303
	Binary Number System	303
	Summary of Applications in Radiology	304
	Questions and Problems	305
24.	DIGITAL X-RAY IMAGING	306
	Introduction	306
	Digital Fluoroscopy	307
	Questions and Problems	312
25.	COMPUTED TOMOGRAPHY	314
	Conventional CT Scanning	314
	Spiral (helical) CT Scanning	319
	Questions and Problems	322
26.	RADIOACTIVITY AND DIAGNOSTIC	
	NUCLEAR MEDICINE	323
	Introduction	323
	Natural Radioactivity	324
	Unstable Atoms	324
	Radioactive Series	324
	Radium	325
	Introduction	325
	Properties	325
	Types of Radiation	325
	Radioactive Decay	326
	Decay Constant	327
	Half-Life	327
	Average Life	327

Radon	328
Radioactive Equilibrium	328
Artificial Radioactivity	329
Isotopes	329
Artificial Radionuclides	329
Nuclear Reactor	330
Nuclear Transformations	332
Properties of Artificial Radionuclides	332
Radioactive Decay	333
Applications of Radionuclides in Medicine	335
Radionuclide Instrumentation	336
Sources of Error in Counting	339
Efficiency and Sensitivity of Counters	341
Geometric Factors in Counting	342
Methods of Counting	343
Important Medical Radionuclides	344
Examples of Radionuclides in Medical Diagnosis	346
Imaging With Radionuclides: the Gamma Camera	348
Collimators	348
Crystal-Photomultiplier Complex	350
Single-Photon Emission Computed Tomography (SPECT)	350
Available Radionuclide Imaging	351
Calibration of Radiopharmaceuticals and Gamma Camera	353
Questions and Problems	353
27. RADIOBIOLOGY	355
Definition	355
History	355
The Physical Basis of Radiobiology	356
Radiobiologic Effects	357
Structure and Function of Cells	358
Nucleus	358
Cytoplasm	359
Cell Reproduction	360
Mitosis	360
Meiosis	362
Structure of DNA	363
Functions of DNA	364

The Radiobiologic Lesion	366
Modes of Action of Ionizing Radiation	366
Cellular Response To Radiation	368
Nuclear Damage	368
Cytoplasmic Damage	369
Cellular Radiosensitivity	369
Modifying Factors in Radiosensitivity	370
Acute Whole-Body Radiation Syndromes	372
Explanation of Acute Whole-Body Radiation Syndromes	374
Dose-Response Models	375
Sigmoid Dose-Response Curve	375
Linear Dose-Response Curve	376
Linear-Quadratic Dose-Response Curve	377
Injurious Effects of Radiation on Normal Tissues	377
Early Effects: Limited Areas of the Body	377
Late Effects	381
Late Somatic Effects in High-Dose Region	382
Late Somatic Effects in Low-Dose Region	383
Risk Estimates for Genetic Damage	386
Radiation Injury to Embryo and Fetus	387
Questions and Problems	390
28. PROTECTION IN RADIOLOGY–HEALTH PHYSICS	391
Introduction	391
Background Radiation	392
Dose Equivalent Limit	394
Derivation of Unit For Risk Assessment	394
Dose Equivalent	395
Numerical Dose Equivalent Limits	396
Occupational	396
Fertile or Pregnant Radiation Workers	398
Nonoccupational (General Public) Limit	398
ALARA Concept	399
Personnel Protection From Exposure To X Rays	400
Protective Measures	402
Protective Barriers in Radiography and Fluoroscopy	403
Working Conditions	404
Acceptance (Compliance) Testing	405

Protection Surveys	406
Protection of the Patient in Diagnostic Radiology	407
Dose Reduction in Radiography	410
Protection in Mammography	414
Computed Tomography Scanning	414
Patient Protection in Fluoroscopy	415
Protection From Electric Shock	416
Protection in Nuclear Medicine	416
Questions and Problems	420
29. NONRADIOLOGIC IMAGING	422
Magnetic Resonance Imaging	422
Nuclear Magnetic Resonance	422
From NMR to MRI	429
Noise in MRI	435
External Field Magnets for MRI	436
Surface Coils	436
A Typical MRI Unit	437
Hazards of MRI	437
Ultrasound Imaging	438
Nature of Sound	438
Properties of Ultrasound	439
Production of Ultrasound	440
Ultrasound Beam Characteristics	441
Echo Reception of Ultrasound	443
Behavior of Ultrasound in Matter	443
Ultrasound Image Displays	446
Question of Biohazard	449
Questions and Problems	449
<i>Appendix—ANSWERS TO PROBLEMS</i>	451
<i>Bibliography</i>	453
<i>Index</i>	457

THE FUNDAMENTALS OF IMAGING PHYSICS AND RADIOBIOLOGY

Chapter 1

SIMPLIFIED MATHEMATICS

Topics Covered in This Chapter

- Arithmetic
 - Fractions
 - Percent
 - Decimal Fractions
 - Significant Figures
- Algebra
 - Ratio and Proportion
 - Plane Geometry
 - Graphs and Charts
 - Large and Small Numbers
 - Logarithms
 - Questions and Problems

ALL OF THE PHYSICAL SCIENCES have in common a firm basis in mathematics. This is no less true of radiologic physics, an important branch of the physical sciences. Clearly, then, in approaching a course in radiologic physics you, as a student technologist, should find your path smoothed by an adequate background in the appropriate areas of mathematics.

We shall assume here that you have had at least the required high school exposure to mathematics, although this may vary widely from place to place. However, realizing that much of this material may have become hazy with time, we shall review the simple but necessary aspects of arithmetic, algebra,

and plane geometry. Such a review should be beneficial in at least two ways. First, it should make it easier to understand the basic principles and concepts of radiologic physics. Second, it should aid in the solution of such everyday problems as conversion of radiographic techniques, interpretation of tube rating charts, determination of radiographic magnification, and many others that may arise from time to time.

The discussion will be subdivided as follows: (1) arithmetic, (2) algebra, (3) ratio and proportion, (4) geometry, (5) graphs and charts, and (6) large and small numbers. Only fundamental principles will be included.

ARITHMETIC

Arithmetic is calculation or problem solving by means of definite numbers. We shall assume that you are familiar with addition, subtraction, multiplication, and division and shall therefore omit these operations.

Fractions

In arithmetic, *a fraction may be defined as one or more equal parts of a unit*. For example, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{2}{5}$ are fractions. The quan-

tity below the line is called the **denominator**; it indicates the number of equal parts into which the unit is divided. The quantity above the line is the **numerator**; it indicates the number of equal parts taken. Thus, if a pie were divided into three equal parts, the denominator would be 3; and if two of these parts were taken, the numerator would be 2, so, the two segments would represent $2/3$ of the pie.

Fractions represent *the division of one quantity by another*. This extends the concept of fractions to expressions in which the numerator is larger than the denominator, as in the fraction $5/2$.

If the numerator is smaller than the denominator, as $3/5$, we have a **proper fraction**. If the numerator is larger than the denominator, as $5/3$, we have an **improper fraction**, because $5 \div 3 = 1^{2/3}$, which is really an integer plus a fraction.

In **adding** fractions, all of which have the **same** denominator, add all the numerators first and then place the sum over the denominator:

$$\frac{2}{7} + \frac{3}{7} + \frac{6}{7} + \frac{5}{7} = \frac{2+3+6+5}{7} = \frac{16}{7}$$

$$\frac{16}{7} = 2\frac{2}{7}$$

Subtraction of fractions having identical denominators follows the same rule:

$$\frac{6}{7} - \frac{4}{7} = \frac{6-4}{7} = \frac{2}{7}$$

If fractions are added or subtracted, and the denominators are **different**, then the **least common denominator** must be found. This is the smallest number which is exactly divisible by all the denominators. Thus,

$$\frac{1}{2} + \frac{2}{3} - \frac{3}{4} = ?$$

The smallest number which is divided exactly by each denominator is 12. Place 12 in the denominator of a new fraction:

$$\frac{\quad}{12}$$

Divide the denominator of each of the fractions in the old equation into 12, and then multiply the answer by the numerator of that fraction; each result is then placed in the numerator of the new fraction:

$$\frac{6+8-9}{12} = \frac{5}{12}$$

Multiplying fractions means taking their product. To multiply fractions, take the product of the numerators and place it over the product of the denominators,

$$\frac{4}{5} \times \frac{3}{10} = \frac{4 \times 3}{5 \times 10} = \frac{12}{50}$$

The resulting fraction can be reduced by dividing the numerator and the denominator by the same number, in this case, 2:

$$\frac{12}{50} \div \frac{2}{2} = \frac{6}{25}$$

which cannot be further simplified.

Note that when the numerator and the denominator are both multiplied or divided by the same number, the value of the fraction does not change. For instance,

$$\frac{3}{5} \times \frac{2}{2} = \frac{6}{10}$$

$$\text{is the same as } \frac{3}{5} \times 1 = \frac{3}{5}$$

When two fractions are to be divided, as $4/5 \div 3/7$, the fraction that is to be divided is the **dividend**, and the fraction that does the dividing is called the **divisor**. In this case, $4/5$ is the dividend and $3/7$ the divisor. The rule is to invert the divisor (called "taking the reciprocal") and multiply the dividend by it:

$$\frac{4}{5} \div \frac{3}{7}$$

$$\frac{4}{5} \times \frac{7}{3} = \frac{28}{15} = 1\frac{13}{15}$$

Percent

A special type of fraction, *percent*, is represented by the sign % to indicate that the number standing with it is to be divided by 100. Thus, $95\% = \frac{95}{100}$. We do not use percentages directly in mathematical computations, but first convert them to fractions or decimals. For instance,

$$\begin{aligned} 150 \times 40\% &\text{ is changed to} \\ 150 \times \frac{40}{100} &\text{ or } 150 \times \frac{2}{5} \\ &\text{ or } 150 \times 0.40. \end{aligned}$$

All these expressions equal 60.

Decimal Fractions

Our common method of representing numbers as multiples of ten is embodied in the *decimal system*. A *decimal fraction* has as its denominator 10, or 10 raised to some power such as 100, 1000, 10,000, etc. The denominator is symbolized by a dot in a certain position. For example, the decimal $0.2 = \frac{2}{10}$; $0.02 = \frac{2}{100}$; $0.002 = \frac{2}{1000}$, etc. Decimals can be multiplied or divided, but care must be taken to place the decimal point in the proper position:

$$\begin{array}{r} 2.24 \\ \times 1.25 \\ \hline 1120 \\ 448 \\ 224 \\ \hline 2.8000 \end{array}$$

First, add the total number of digits to the right of the decimal points in the numbers being multiplied, which in this case turns out to be four. Then point off four places from the right in the answer to determine the correct position of the decimal point. The decimal system is used everywhere in science and in the vast majority of countries in daily life.



Figure 1.1. With this calibrated scale we can estimate to the nearest tenth. Thus, the position of the pointer indicates 8.4 units, the 0.4 being the last significant figure.

Significant Figures

The precision (reproducibility of results) of any type of measurement is limited by the design of the measuring instrument. For example, a scale calibrated in grams as shown in Figure 1.1 allows an estimate to the nearest tenth of a gram. Thus, the scale in Figure 1.1 reads 8.4 grams. The last figure, 0.4, is estimated and is the *last significant figure*—that is, it is the last meaningful digit. Obviously, no greater precision is possible with this particular instrument. To improve precision, the scale would have to show a greater number of subdivisions.

Significant figures are used in various mathematical operations. For example, in addition:

item 1	98.26	grams
item 2	1.350	g
item 3	<u>260.1</u>	g
	359.710	g

Notice that three digits appear after the decimal point in the answer. But in item (3) there is only one digit, 1, after the decimal point; beyond this, the digits are unknown. Therefore, the digits after the 7 in the answer imply more than is known, since the answer can be no more precise than the least precise item being added. In this case, the answer should be properly stated as 359.7. In addition and subtraction the answer can have no more significant figures after the decimal point than the item with the *least number of significant figures after its decimal point*.

A different situation exists in multiplica-

tion and division. Here, the total number of significant figures in the answer equals that in the items having the **least total number of significant figures**. For example, in

$$\begin{array}{r} 25.23 \text{ cm} \\ \times 1.21 \text{ cm} \\ \hline 2523 \\ 5046 \\ \hline 2523 \\ 30.5283 \text{ cm}^2 \end{array}$$

1.21 has fewer significant figures—three in all—so the answer should have three significant figures and be read as 30.5 (dropping

the 0.0283).

In general, to **round off** significant figures, observe the following rule: if the digit following the last significant figure is equal to or greater than 5, the last significant figure is increased by 1; if less than 5, it is unchanged. The rule is applied in the following examples:

$$\begin{array}{l} 45.157 \text{ is rounded to } 45.16 \\ 45.155 \text{ is rounded to } 45.16 \\ 45.153 \text{ is rounded to } 45.15 \end{array}$$

where the answer is to be expressed in four significant figures.

ALGEBRA

The word **algebra**, derived from the Arabic language, connotes that branch of mathematics which deals with the relationship of quantities usually represented by letters of the alphabet—Roman, Greek, or Hebrew.

Operations. Mathematical operations with **letter symbols** are the same as with **numerals**, since both are symbolic representations of numbers which, in themselves, are abstract concepts. For example, the concept “four” may be represented by 4, 2^2 , 2×2 , $2 + 2$, or $3 + 1$; or by the letter x if the value of x is specifically designated to represent “four.” In algebra, just as in arithmetic, the fundamental operations include addition, subtraction, multiplication, and division; and there are fractions, proportions, and equations. Algebra provides a method of finding an unknown quantity when the relationship of certain known quantities is specified.

Algebraic operations are indicated by the same symbols as in arithmetic:

$$\begin{array}{ll} + & \text{(plus) add} \\ - & \text{(minus) subtract} \\ \times & \text{(times) multiply} \\ \div & \text{(divided by) divide} \end{array}$$

= equals

To indicate **addition** in algebra, use the general expression

$$x + y \quad (1)$$

The symbols x and y , called **variables**, may represent any number or quantity. Thus, if $x = 4$, and $y = 7$, then, substituting these values in equation (1),

$$4 + 7 = 11$$

Similarly, to indicate **subtraction** in algebra, use the general expression

$$x - y$$

If $x = 9$ and $y = 5$ then

$$9 - 5 = 4$$

Notice that algebraic symbols may represent whole numbers, fractions, zero, and negative numbers, among others. Negative numbers are those whose value is less than zero and are designated as $-x$. In algebraic terms, add a positive and a negative number as follows:

$$x + -y$$